LESSON

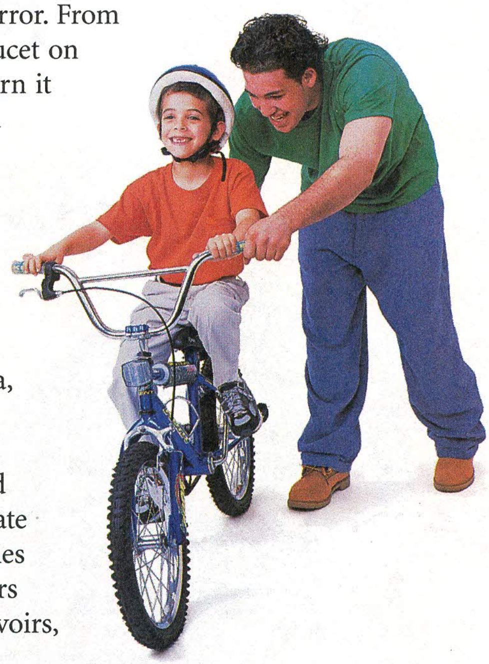
2.1

Inductive Reasoning

We have to reinvent the wheel every once in a while, not because we need a lot of wheels; but because we need a lot of inventors.

BRUCE JOYCE

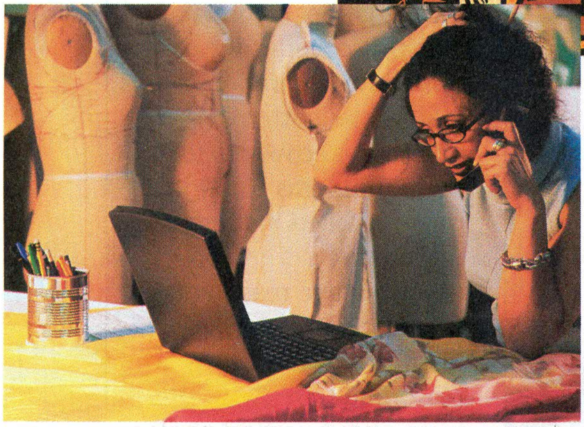
As a child you learned by experimenting with the natural world around you. You learned how to walk, to talk, and to ride your first bicycle, all by trial and error. From experience you learned to turn a water faucet on with a counterclockwise motion and to turn it off with a clockwise motion. You achieved most of your learning by a process called **inductive reasoning.** It is the process of observing data, recognizing patterns, and making generalizations about those patterns.



Geometry is rooted in inductive reasoning. In ancient Egypt and Babylonia, geometry began when people developed procedures for measurement after much experience and observation. Assessors and surveyors used these procedures to calculate land areas and to reestablish the boundaries of agricultural fields after floods. Engineers used the procedures to build canals, reservoirs, and the Great Pyramids. Throughout this course you will use inductive reasoning. You will perform investigations, observe similarities and patterns, and make many discoveries that you can use to solve problems.

Language CONNECTION

The word "geometry" means "measure of the earth" and was originally inspired by the ancient Egyptians. The ancient Egyptians devised a complex system of land surveying in order to reestablish land boundaries that were erased each spring by the annual flooding of the Nile River.



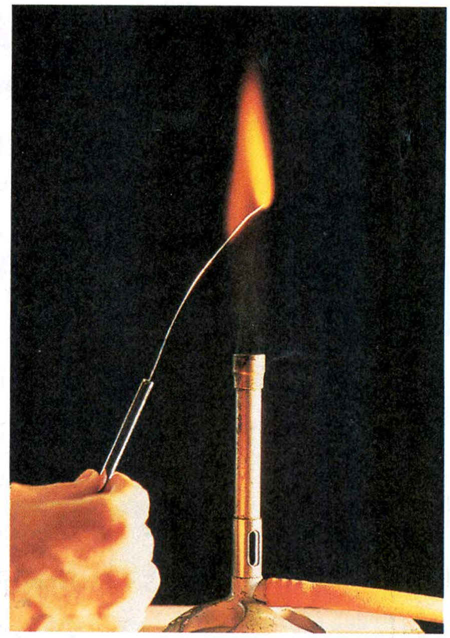
Inductive reasoning guides scientists, investors, and business managers. All of these professionals use past experience to assess what is likely to happen in the future.

When you use inductive reasoning to make a generalization, the generalization is called a **conjecture.** Consider the following example from science.

EXAMPLE A

A scientist dips a platinum wire into a solution containing salt (sodium chloride), passes the wire over a flame, and observes that it produces an orange-yellow flame.

She does this with many other solutions that contain salt, finding that they all produce an orange-yellow flame. Make a conjecture based on her findings.



Platinum wire flame test

▶ Solution

The scientist tested many other solutions containing salt, and found no counterexamples. You should conjecture: "If a solution contains sodium chloride, then in a flame test it produces an orange-yellow flame."

Like scientists, mathematicians often use inductive reasoning to make discoveries. For example, a mathematician might use inductive reasoning to find patterns in a number sequence. Once he knows the pattern, he can find the next term.

EXAMPLE B

Consider the sequence

2, 4, 7, 11, . . .

Make a conjecture about the rule for generating the sequence. Then find the next three terms.

▶ Solution

Look at the numbers you add to get each term. The 1st term in the sequence is 2. You add 2 to find the 2nd term. Then you add 3 to find the 3rd term, and so on.

$$2 \xrightarrow{+2} 4 \xrightarrow{+3} 7 \xrightarrow{+4} 11$$

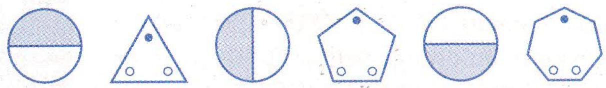
You can conjecture that if the pattern continues, you always add the next counting number to get the next term. The next three terms in the sequence will be 16, 22, and 29.

$$11 \xrightarrow{+5} 16 \xrightarrow{+6} 22 \xrightarrow{+7} 29$$

In the following investigation you will use inductive reasoning to recognize a pattern in a series of drawings and use it to find a term much farther out in a sequence.

Investigation
Shape Shifters

Look at the sequence of shapes below. Pay close attention to the patterns that occur in every other shape.



Step 1 What patterns do you notice in the 1st, 3rd, and 5th shapes?

Step 2 What patterns do you notice in the 2nd, 4th, and 6th shapes?

Step 3 Draw the next two shapes in the sequence.

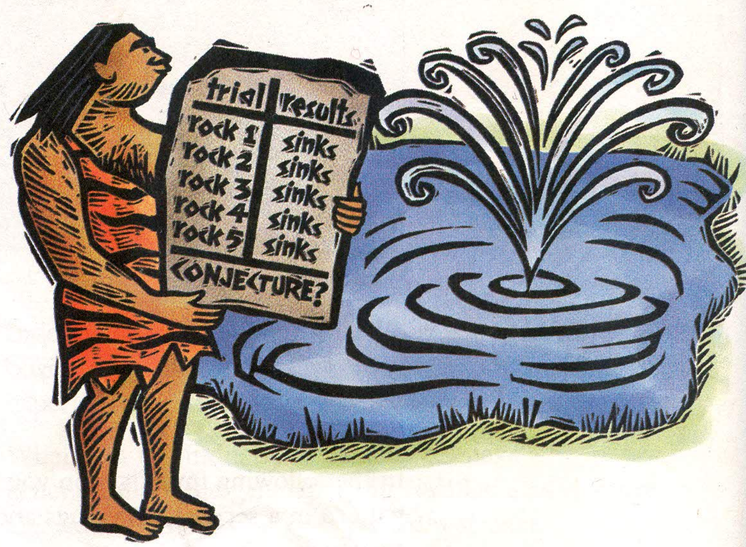
Step 4 Use the patterns you discovered to draw the 25th shape.

Step 5 Describe the 30th shape in the sequence. You do not have to draw it!

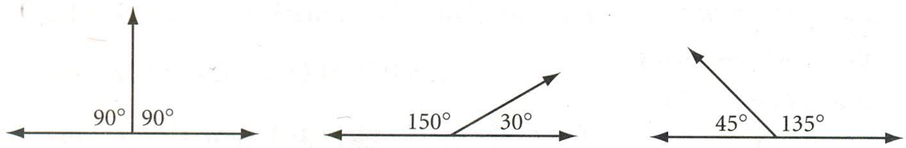
Sometimes a conjecture is difficult to find because the data collected are unorganized or the observer is mistaking coincidence with cause and effect. Good use of inductive reasoning depends on the quantity and quality of data. Sometimes not enough information or data have been collected to make a proper conjecture. For example, if you are asked to find the next term in the pattern 3, 5, 7, you might conjecture that the next term is 9—the next odd number. Someone else might notice that the pattern is the consecutive odd primes and say that the next term is 11. If the pattern was 3, 5, 7, 11, 13, what would you be more likely to conjecture?

Exercises

1. On his way to the local Hunting and Gathering Convention, caveperson Stony Grok picks up a rock, drops it into a lake, and notices that it sinks. He picks up a second rock, drops it into the lake, and notices that it also sinks. He does this five more times. Each time, the rock sinks straight to the bottom of the lake. Stony conjectures: "Ura nok seblu," which translates to __?__. What counterexample would Stony Grok need to find to disprove, or at least to refine, his conjecture? (h)



2. Sean draws these geometric figures on paper. His sister Courtney measures each angle with a protractor. They add the measures of each pair of angles to form a conjecture. Write their conjecture.



For Exercises 3–10, use inductive reasoning to find the next two terms in each sequence.

3. 1, 10, 100, 1000, ?, ?

4. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, ?, ? (h)

5. 7, 3, −1, −5, −9, −13, ?, ?

6. 1, 3, 6, 10, 15, 21, ?, ?

7. 1, 1, 2, 3, 5, 8, 13, ?, ? (h)

8. 1, 4, 9, 16, 25, 36, ?, ? (h)

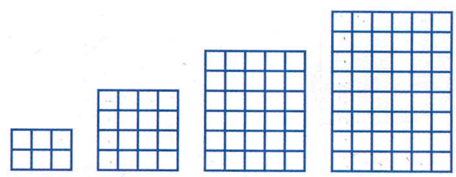
9. 32, 30, 26, 20, 12, 2, ?, ?

10. 1, 2, 4, 8, 16, 32, ?, ?

For Exercises 11–16, use inductive reasoning to draw the next shape in each picture pattern.

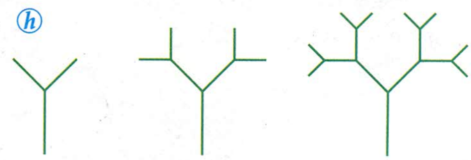
11.

12.



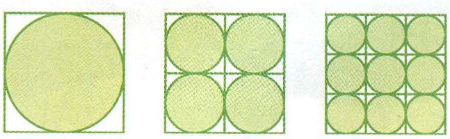
13. (h)

14. (h)



15. (h)

16.



Use the rule provided to generate the first five terms of the sequence in Exercise 17 and the next five terms of the sequence in Exercise 18.

17. $3n - 2$ (h)

18. $1, 3, 6, 10, \ldots, \frac{n(n+1)}{2}, \ldots$

19. Now it's your turn. Generate the first five terms of a sequence. Give the sequence to a member of your family or to a friend and ask him or her to find the next two terms in the sequence. Can he or she find your pattern?

20. Write the first five terms of two different sequences in which 12 is the 3rd term.

21. Think of a situation in which you have used inductive reasoning. Write a paragraph describing what happened and explaining why you think it was inductive reasoning. (h)

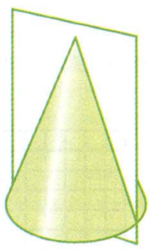
22. Look at the pattern in these pairs of equations. Decide if the conjecture is true. If it is not true, find a counterexample.

$$12^2 = 144 \quad \text{and} \quad 21^2 = 441$$
$$13^2 = 169 \quad \text{and} \quad 31^2 = 961$$
$$103^2 = 10609 \quad \text{and} \quad 301^2 = 90601$$
$$112^2 = 12544 \quad \text{and} \quad 211^2 = 44521$$

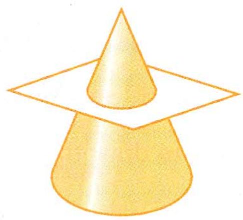
Conjecture: If two numbers have the same digits in reverse order, then the squares of those numbers will have identical digits but in reverse order.

Review

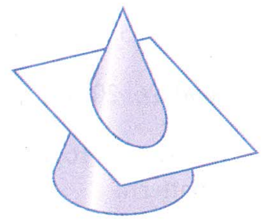
23. Sketch the section formed when the cone is sliced by the plane, as shown.



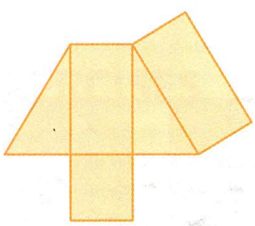
24.



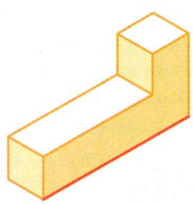
25.



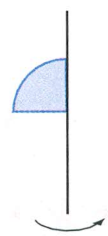
26. Sketch the three-dimensional figure formed by folding the net below into a solid. ⓗ



27. Sketch the figure shown below but with the red edge vertical and facing you. ⓗ



28. Sketch the solid of revolution formed when the two-dimensional figure is rotated about the line. ⓗ



For Exercises 29–38, write the word that makes the statement true.

29. Points are ? if they lie on the same line.

30. A triangle with two congruent sides is ?.

31. The geometry tool used to measure the size of an angle in degrees is called a(n) ?.

32. A(n) ? of a circle connects its center to a point on the circle.

33. A segment connecting any two non-adjacent vertices in a polygon is called a(n) ?.

34. A polygon with 12 sides is called a(n) ?.

35. A trapezoid has exactly one pair of ? sides.

36. A(n) _?_ polygon is both equiangular and equilateral.

37. If angles are complementary, then their measures add to _?_.

38. If two lines intersect to form a right angle, then they are _?_.

For Exercises 39–42, sketch and label the figure.

39. Pentagon *GIANT* with diagonal $\overline{AG}$ parallel to side $\overline{NT}$

40. A quadrilateral that has reflectional symmetry but not rotational symmetry

41. A prism with a hexagonal base

42. A counterexample to show that the following statement is false: The diagonals of a kite bisect the angles. ⓗ

IMPROVING YOUR REASONING SKILLS

Puzzling Patterns

These patterns are "different." Your task is to find the next term.

1. 18, 46, 94, 63, 52, 61, _?_
2. O, T, T, F, F, S, S, E, N, _?_
3. 1, 4, 3, 16, 5, 36, 7, _?_
4. 4, 8, 61, 221, 244, 884, _?_
5. 6, 8, 5, 10, 3, 14, 1, _?_
6. **B, 0, C, 2, D, 0, E, 3, F, 3, G,** _?_
7. 2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 3, 2, 3, 2, 3, _?_
8. A E F H I K L M N T V W
   B C D G J O P Q R S U
   Where do the X, Y, and Z go?

LESSON

2.2

Deductive Reasoning

That's the way things come clear. All of a sudden. And then you realize how obvious they've been all along.

MADELEINE L'ENGLE

Have you ever noticed that the days are longer in the summer? Or that mosquitoes appear after a summer rain? Over the years you have made conjectures, using inductive reasoning, based on patterns you have observed. When you make a conjecture, the process of discovery may not always help explain *why* the conjecture works. You need another kind of reasoning to help answer this question.

Deductive reasoning is the process of showing that certain statements follow logically from agreed-upon assumptions and proven facts. When you use deductive reasoning, you try to reason in an orderly way to convince yourself or someone else that your conclusion is valid. If your initial statements are true, and you give a logical argument, then you have shown that your conclusion is true. For example, in a trial, lawyers use deductive arguments to show how the evidence that they present proves their case. A lawyer might make a very good argument. But first, the court must believe the evidence and accept it as true.

The success of an attorney's case depends on the jury accepting the evidence as true and following the steps in her deductive reasoning.

You use deductive reasoning in algebra. When you provide a reason for each step in the process of solving an equation, you are using deductive reasoning. Here is an example.

EXAMPLE A

Solve the equation for x. Give a reason for each step in the process.

$$3(2x + 1) + 2(2x + 1) + 7 = 42 - 5x$$

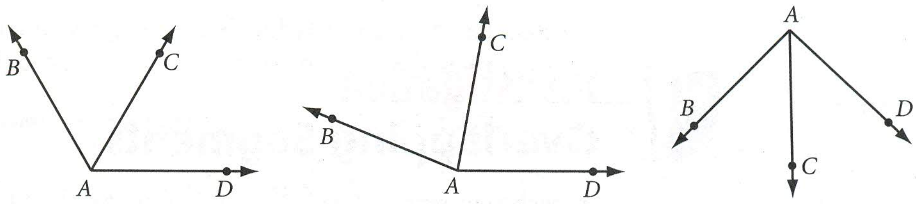
▶ Solution

$3(2x + 1) + 2(2x + 1) + 7 = 42 - 5x$	The original equation.
$5(2x + 1) + 7 = 42 - 5x$	Combining like terms.
$5(2x + 1) = 35 - 5x$	Subtraction property of equality.
$10x + 5 = 35 - 5x$	Distributive property.
$10x = 30 - 5x$	Subtraction property of equality.
$15x = 30$	Addition property of equality.
$x = 2$	Division property of equality.

The next example shows how to use both kinds of reasoning: inductive reasoning to discover the property and deductive reasoning to explain why it works.

EXAMPLE B

In each diagram, $\overrightarrow{AC}$ bisects obtuse angle BAD. Classify $\angle BAD$, $\angle DAC$, and $\angle CAB$ as *acute, right,* or *obtuse*. Then complete the conjecture.



$m\angle BAD = 120°$ $m\angle BAD = 158°$ $m\angle BAD = 92°$

Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles are $\underline{\ ?\ }$.

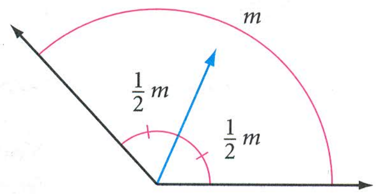
Justify your answers with a deductive argument.

► Solution

In each diagram, $\angle BAD$ is obtuse because $m\angle BAD$ is greater than 90°. In each diagram, the angles formed by the bisector are acute because their measures—60°, 79°, and 46°—are less than 90°. So one possible conjecture is

Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles are *acute.*

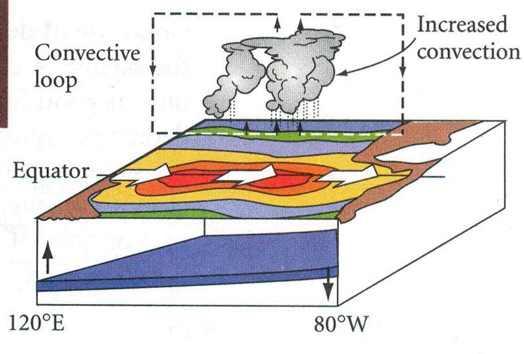
Why? According to our definition of an angle, every angle measure is less than 180°. So, using algebra, if m is the measure of an obtuse angle, then $m < 180°$. When you bisect an angle, the two newly formed angles each measure half of the original angle, or $\frac{1}{2}m$. If $m < 180°$, then $\frac{1}{2}m < \frac{1}{2}(180)$, so $\frac{1}{2}m < 90°$. The two angles are each less than 90°, so they are acute.



Science CONNECTION

Here is an example of inductive reasoning, supported by deductive reasoning. El Niño is the warming of water in the tropical Pacific Ocean, which produces unusual weather conditions and storms worldwide. For centuries, farmers living in the Andes Mountains of South America have observed the stars in the Pleiades constellation to predict El Niño conditions. If the Pleiades look dim in June, they predict an El Niño year. What is the connection? Scientists have recently found that in an El Niño year, increased evaporation from the ocean produces high-altitude clouds that are invisible to the eye, but create a haze that makes stars more difficult to see. Therefore, the pattern that the Andean farmers knew about for centuries is now supported by a scientific explanation. To find out more about this story, go to www.keymath.com/DG .

El Niño Conditions



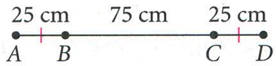
Inductive reasoning allows you to discover new ideas based on observed patterns. Deductive reasoning can help explain why your conjectures are true.

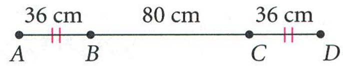
Inductive and deductive reasoning work very well together. In this investigation you will use inductive reasoning to form a conjecture and deductive reasoning to explain why it's true.

Investigation

Overlapping Segments

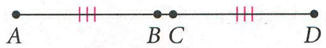
In each segment, $\overline{AB} \cong \overline{CD}$.





Step 1 From the markings on each diagram, determine the lengths of $\overline{AC}$ and $\overline{BD}$. What do you discover about these segments?

Step 2 Draw a new segment. Label it $\overline{AD}$. Place your own points B and C on $\overline{AD}$ so that $\overline{AB} \cong \overline{CD}$.



Step 3 Measure $\overline{AC}$ and $\overline{BD}$. How do these lengths compare?

Step 4 Complete the conclusion of this conjecture:

If $\overline{AD}$ has points A, B, C, and D in that order with $\overline{AB} \cong \overline{CD}$, then __?__.

Now you will use deductive reasoning and algebra to explain why the conjecture from Step 4 is true.

Step 5 Use deductive reasoning to convince your group that AC will always equal BD. Take turns explaining to each other. Write your argument algebraically.

In the investigation you used both inductive and deductive reasoning to convince yourself of the overlapping segments property. You will use a similar process in the next lesson to discover and prove the overlapping angles property in Exercise 17.

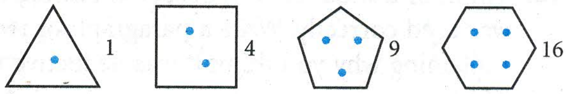
Good use of deductive reasoning depends on the quality of the argument. Just like the saying, "A chain is only as strong as its weakest link," a deductive argument is only as good (or as true) as the statements used in the argument. A conclusion in a deductive argument is true only if *all* the statements in the argument are true. Also, the statements in your argument must clearly follow from each other. Did you use clear arguments in explaining the investigation steps? Did you point out that $\overline{BC}$ is part of both $\overline{AC}$ and $\overline{BD}$? Did you point out that if you add the same amount to things that are equal the resulting sum must be equal?

EXERCISES

1. When you use _?_ reasoning you are generalizing from careful observation that something is probably true. When you use _?_ reasoning you are establishing that, if a set of properties is accepted as true, something else must be true.

2. $\angle A$ and $\angle B$ are complementary. $m\angle A = 25°$. What is $m\angle B$? What type of reasoning do you use, inductive or deductive reasoning, when solving this problem?

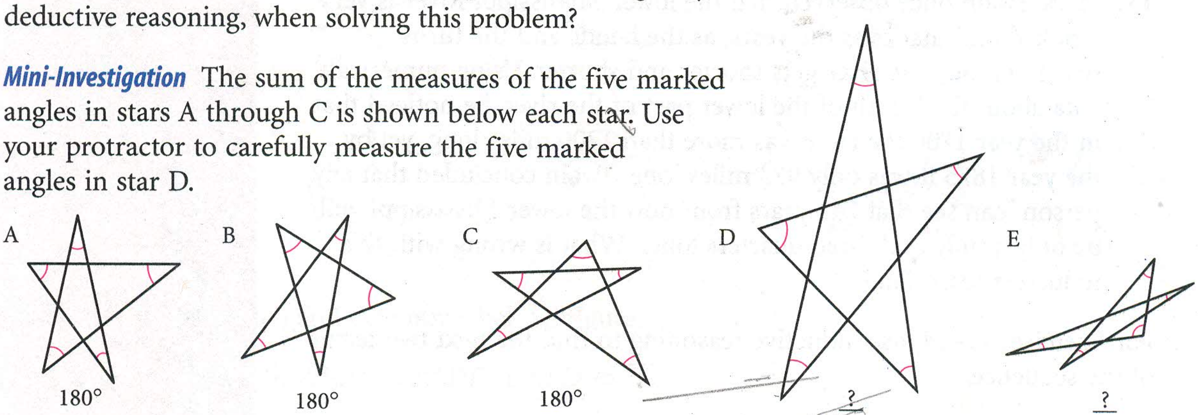
3. If the pattern continues, what are the next two terms?



What type of reasoning do you use, inductive or deductive reasoning, when solving this problem?

4. $\triangle DGT$ is isosceles with $TD = DG$. If the perimeter of $\triangle DGT$ is 756 cm and $GT = 240$ cm, then $DG =$ _?_. What type of reasoning do you use, inductive or deductive reasoning, when solving this problem?

5. ***Mini-Investigation*** The sum of the measures of the five marked angles in stars A through C is shown below each star. Use your protractor to carefully measure the five marked angles in star D.



If this pattern continues, without measuring, what would be the sum of the measures of the marked angles in star E? What type of reasoning do you use, inductive or deductive reasoning, when solving this problem?

6. The definition of a parallelogram says, "If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram." Quadrilateral $LNDA$ has both pairs of opposite sides parallel. What conclusion can you make? What type of reasoning did you use?

7. Use the overlapping segments property to complete each statement.

a. If $AB = 3$, then $CD =$ _?_.
b. If $AC = 10$, then $BD =$ _?_.
c. If $BC = 4$ and $CD = 3$, then $AC =$ _?_.

8. In Example B of this lesson you discovered through inductive reasoning that if an obtuse angle is bisected, then the two newly formed congruent angles are acute. You then used deductive reasoning to explain why they were acute. Go back to the example and look at the sizes of the acute angles formed. What is the smallest possible size for the two congruent acute angles formed by the bisector? Can you use deductive reasoning to explain why? ⓗ

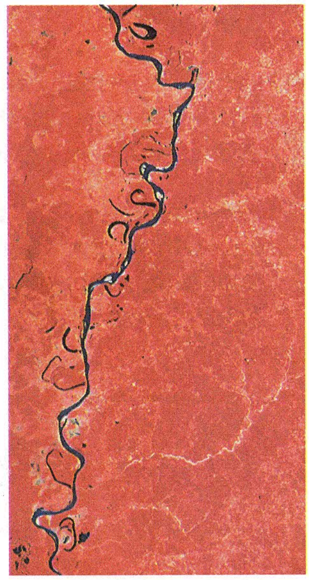
9. Study the pattern and make a conjecture by completing the fifth line. What would be the conjecture for the sixth line? The tenth line? (h)

$$
\begin{aligned}
1 \cdot 1 &= 1\\
11 \cdot 11 &= 121\\
111 \cdot 111 &= 12{,}321\\
1{,}111 \cdot 1{,}111 &= 1{,}234{,}321\\
11{,}111 \cdot 11{,}111 &= \underline{\ ?\ }
\end{aligned}
$$

10. Think of a situation you observed outside of school in which deductive reasoning was used correctly. Write a paragraph or two describing what happened and explaining why you think it was deductive reasoning.

Review

11. Mark Twain once observed that the lower Mississippi River is very crooked and that over the years, as the bends and the turns straighten out, the river gets shorter and shorter. Using numerical data about the length of the lower part of the river, he noticed that in the year 1700 the river was more than 1200 miles long, yet by the year 1875 it was only 973 miles long. Twain concluded that any person "can see that 742 years from now the lower Mississippi will be only a mile and three-quarters long." What is wrong with this inductive reasoning?



Aerial photo of the Mississippi River

For Exercises 12–14, use inductive reasoning to find the next two terms of the sequence.

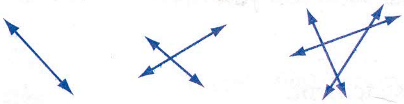
12. 180, 360, 540, 720, ?, ? (h)

13. 0, 10, 21, 33, 46, 60, ?, ?

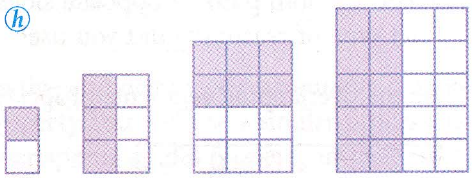
14. $\frac{1}{2}$, 9, $\frac{2}{3}$, 10, $\frac{3}{4}$, 11, ?, ?

For Exercises 15–18, draw the next shape in each picture pattern.

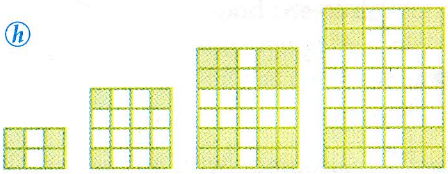
15.



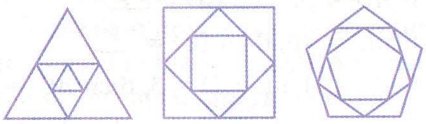
16. (h)



17. (h)



18.



19. Think of a situation you have observed in which inductive reasoning was used incorrectly. Write a paragraph or two describing what happened and explaining why you think it was an incorrect use of inductive reasoning.

Match each term in Exercises 20–29 with one of the figures A–O.

20. Kite

21. Consecutive angles in a polygon

22. Trapezoid

23. Diagonal in a polygon

24. Pair of complementary angles

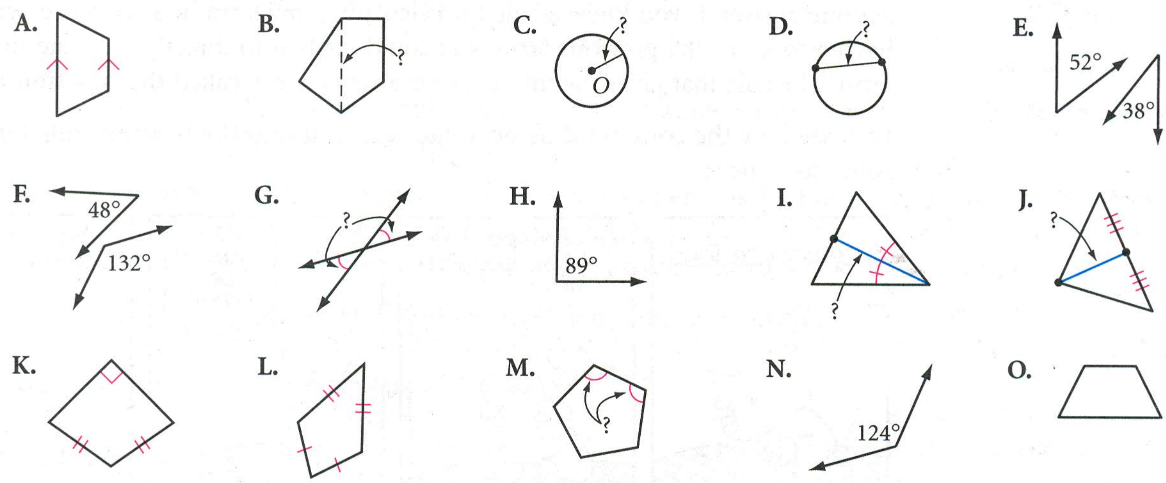
25. Radius

26. Pair of vertical angles

27. Chord

28. Acute angle

29. Angle bisector in a triangle



For Exercises 30–33, sketch and carefully label the figure.

30. Pentagon $WILDE$ with $\angle ILD \cong \angle LDE$ and $\overline{LD} \cong \overline{DE}$

31. Isosceles obtuse triangle OBG with $m\angle BGO = 140°$

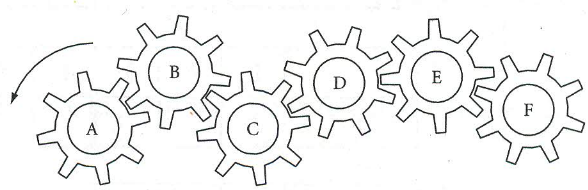
32. Circle O with a chord $\overline{CD}$ perpendicular to radius $\overline{OT}$

33. Circle K with inscribed angle DIN

IMPROVING YOUR VISUAL THINKING SKILLS

Rotating Gears

In what direction will gear E rotate if gear A rotates in a counterclockwise direction?



LESSON 2.3

Finding the *n*th Term

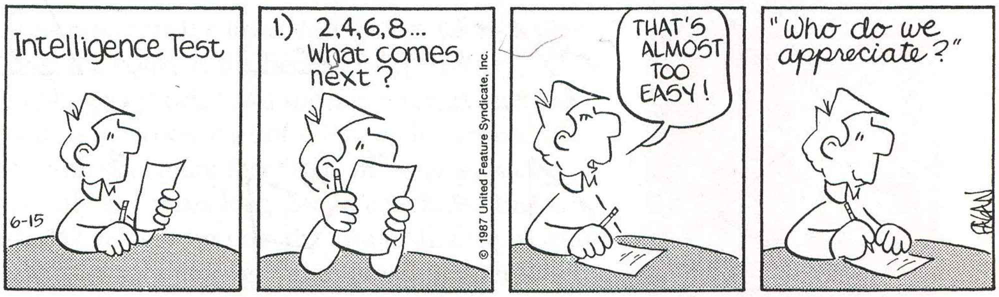
If you do something once, people call it an accident. If you do it twice, they call it a coincidence. But do it a third time and you've just proven a natural law.

GRACE MURRAY HOPPER

What would you do to get the next term in the sequence 20, 27, 34, 41, 48, 55, . . . ? A good strategy would be to find a pattern, using inductive reasoning. Then you would look at the differences between consecutive terms and predict what comes next. In this case there is a constant difference of +7. That is, you add 7 each time.

The next term is 55 + 7, or 62. What if you needed to know the value of the 200th term of the sequence? You certainly don't want to generate the next 193 terms just to get one answer. If you knew a rule for calculating *any* term in a sequence, without having to know the previous term, you could apply it to directly calculate the 200th term. The rule that gives the *n*th term for a sequence is called the **function rule.**

Let's see how the constant difference can help you find the function rule for some sequences.



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Investigation
Finding the Rule

Step 1 Copy and complete each table. Find the differences between consecutive values.

a.

n	1	2	3	4	5	6	7	8
$n - 5$	-4	-3	-2					

b.

n	1	2	3	4	5	6	7	8
$4n - 3$	1	5	9					

c.

n	1	2	3	4	5	6	7	8
$-2n + 5$	3	1	-1					

d.

n	1	2	3	4	5	6	7	8
$3n - 2$	1	4	7					

e.

n	1	2	3	4	5	6	7	8
$-5n + 7$	2	-3	-8					

Step 2 | Did you spot the pattern? If a sequence has a constant difference of 4, then the number in front of the n (the coefficient of n) is $\underline{\ ?\ }$. In general, if the difference between the values of consecutive terms of a sequence is always the same, say m (a constant), then the coefficient of n in the formula is $\underline{\ ?\ }$.

Let's return to the sequence at the beginning of the lesson.

Term	1	2	3	4	5	6	7	...	n
Value	20	27	34	41	48	55	62	...	

+7 +7

The constant difference is 7, so you know part of the rule is $7n$. How do you find the rest of the rule?

Step 3 | The first term ($n = 1$) of the sequence is 20, but if you apply the part of the rule you have so far using $n = 1$, you get $7n = 7(1) = 7$, not 20. So how should you fix the rule? How can you get from 7 to 20? What is the rule for this sequence?

Step 4 | Check your rule by trying the rule with other terms in the sequence.

Let's look at an example of how to find a function rule, for the nth term in a number pattern.

EXAMPLE A | Find the rule for the sequence 7, 2, −3, −8, −13, −18, . . .

▸ Solution | Placing the terms and values in a table we get

Term	1	2	3	4	5	6	...	n
Value	7	2	−3	−8	−13	−18	...	

The difference between the terms is always −5. So the rule is

$-5n +$ "something"

Let's use c to stand for the unknown "something." So the rule is

$5n + c$

To find c, replace the n in the rule with a term number. Try $n = 1$ and set the expression equal to 7.

$$-5(1) + c = 7$$
$$c = 12$$

The rule is $-5n + 12$.

You can find the value of any term in the sequence by substituting the term number for n in the function rule. Let's look at an example of how to find the 200th term in a geometric pattern.

EXAMPLE B

If you place 200 points on a line, into how many non-overlapping rays and segments does it divide the line?

▶ Solution

Wait! don't start placing 200 points on a line. You need to find a rule that relates the number of points placed on a line to the number of parts created by those points. Then you can use your rule to answer the problem.

Start by creating a table.

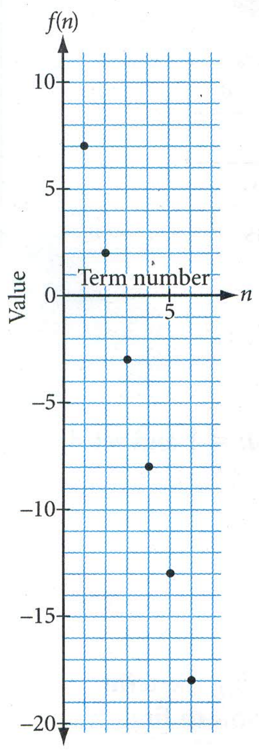
Points dividing the line	1	2	3	4	5	6	...	n	...	200
Non-overlapping rays							...		...	
Non-overlapping segments							...		...	
Total							...		...	

Sketch one point dividing a line. One point gives you just two rays. Enter that into the table.

Next, sketch two points dividing a line. This gives one segment and the two end rays. Enter the value into your table.

Next, sketch three points dividing a line, then four, then five, and so on. The table completed for one to three points is

Points dividing the line	1	2	3	4	5	6	...	n	...	200
Non-overlapping rays	2	2	2				...		...	
Non-overlapping segments	0	1	2				...		...	
Total	2	3	4				...		...	



Once you have found values for 1, 2, 3, 4, 5, and 6 points on a line you next try to find the rule for each sequence. There are always two non-overlapping rays so for 200 points there will be two rays. The rule, or nth term, for the number of non-overlapping segments is $n - 1$. For 200 points there will be 199 segments. The rule, or nth term, for the total number of distinct rays and segments of the line is $n + 1$. For 200 points there will be 201 distinct parts of the line.

This process of looking at patterns and generalizing a rule, or nth term, is inductive reasoning. To understand why the rule is what it is, you can turn to deductive reasoning. Notice that adding another point on a line divides a segment into two segments. So each new point adds one more segment to the pattern.

Rules that generate a sequence with a constant difference are **linear functions.** To see why they're called linear, you can graph the term number and the value for the sequence as ordered pairs of the form (*term number, value*) on the coordinate plane. At left is the graph of the sequence from Example A.

Term number n	1	2	3	4	5	6	...	n
Value $f(n)$	7	2	−3	−8	−13	−18	...	$-5n + 12$

EXERCISES

For Exercises 1–3, find the function rule $f(n)$ for each sequence. Then find the 20th term in the sequence.

1.

n	1	2	3	4	5	6	...	n	...	20
$f(n)$	3	9	15	21	27	33	...		...	

2.

n	1	2	3	4	5	6	...	n	...	20
$f(n)$	1	-2	-5	-8	-11	-14	...		...	

3.

n	1	2	3	4	5	6	...	n	...	20
$f(n)$	-4	4	12	20	28	36	...		...	

For Exercises 4–6, find the rule for the nth figure. Then find the number of colored tiles or matchsticks in the 200th figure.

4.

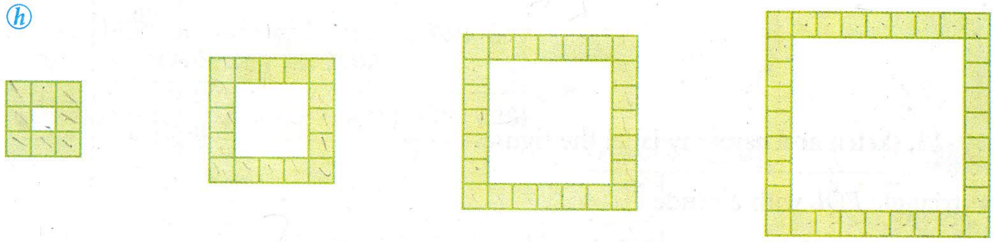


Figure number	1	2	3	4	5	6	...	n	...	200
Number of tiles	8						...		...	

5.

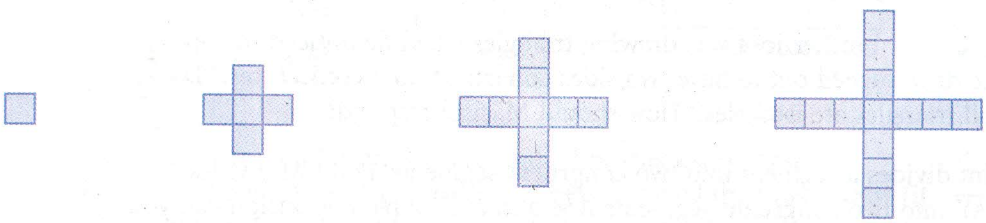


Figure number	1	2	3	4	5	6	...	n	...	200
Number of tiles		5					...		...	

6.

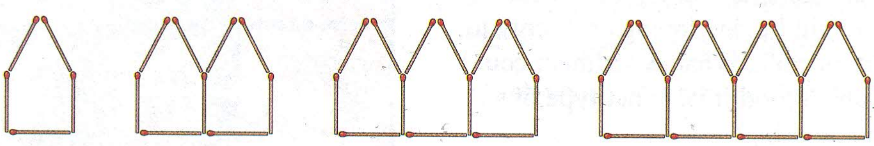
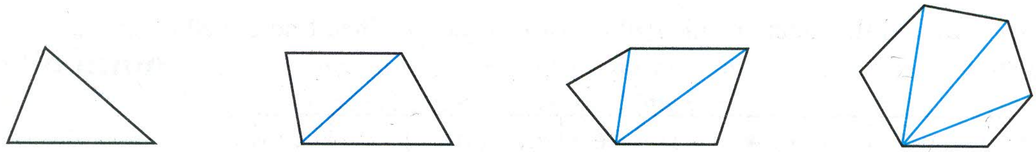


Figure number	1	2	3	4	5	6	...	n	...	200
Number of matchsticks	5	9					...		...	
Number of matchsticks in perimeter of figure	5	8					...		...	

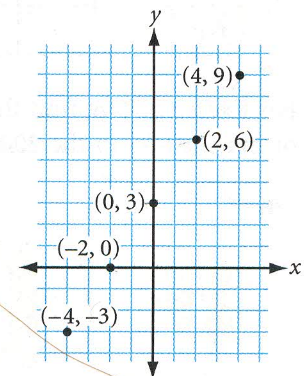
7. How many triangles are formed when you draw all the possible diagonals from just one vertex of a 35-gon? ⓗ



Number of sides	3	4	5	6	...	n	...	35
Number of triangles formed	1	2	3		...		...	

8. Graph the values in your tables from Exercises 4–6. Which set of points lies on a steeper line? What number in the rule gives a measure of steepness?

9. Find the rule for the set of points in the graph shown at right. Place the x-coordinate of each ordered pair in the top row of your table and the corresponding y-coordinate in the second row. What is the value of y in terms of x?



Review

For Exercises 10–13, sketch and carefully label the figure.

10. Equilateral triangle EQL with altitude $\overline{QT}$

11. Isosceles obtuse triangle OLY with $\overline{OL} \cong \overline{YL}$ and angle bisector $\overline{LM}$

12. A cube with a plane passing through it; the cross section is rectangle $RECT$

13. A net for a rectangular solid with the dimensions 1 by 2 by 3 cm

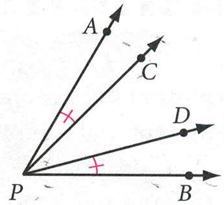
14. Márisol's younger brother José was drawing triangles when he noticed that every triangle he drew turned out to have two sides congruent. José conjectures: "Look, Márisol, all triangles are isosceles." How should Márisol respond?

15. A midpoint divides a segment into two congruent segments. Point M divides segment $\overline{AY}$ into two congruent segments $\overline{AM}$ and $\overline{MY}$. What conclusion can you make? What type of reasoning did you use?

16. Tanya's favorite lunch is peanut butter and jelly on wheat bread with a glass of milk. Lately, she has been getting an allergic reaction after eating this lunch. She is wondering if she might be developing an allergy to peanut butter, wheat, or milk. What experiment could she do to find out which food it is? What type of reasoning would she be using?

17. ***Mini-Investigation*** Do the geometry investigation and make a conjecture.

Given $\angle APB$ with points C and D in its interior and $m\angle APC = m\angle DPB$,



If $m\angle APD = 48°$, then $m\angle CPB = \underline{?}$

If $m\angle CPB = 17°$, then $m\angle APD = \underline{?}$

If $m\angle APD = 62°$, then $m\angle CPB = \underline{?}$

Conjecture: If points C and D lie in the interior of $\angle APB$, and $m\angle APC = m\angle DPB$ then $m\angle APD = \underline{?}$ (Overlapping angles property)

project

BEST-FIT LINES

The following table and graph show the mileage and lowest priced round-trip airfare between New York City and each destination city. Is there a relationship between the money you spend and how far you can travel?

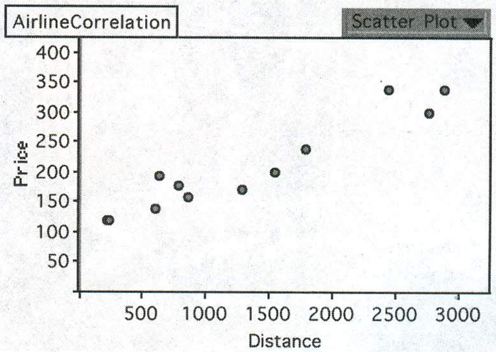
Fathom™

With Fathom Dynamic Statistics™ software, you can plot your data points and find the linear equation that best fits your data.

Lowest Round-trip Airfares from New York City on February 25, 2002

Destination City	Distance (miles)	Price ($)
Boston	215	$118
Chicago	784	$178
Atlanta	865	$158
Miami	1286	$170
Denver	1791	$238
Phoenix	2431	$338
Los Angeles	2763	$298

Source: http://www.Expedia.com



Even though the data are not linear, you can find a linear equation that *approximately* fits the data. The graph of this equation is called the **line of best fit.** How would you use the line of best fit to predict the cost of a round-trip ticket to Seattle (2814 miles)? How would you use it to determine how far you could travel (in miles) with $250? How accurate do you think the answer would be?

Choose a topic and a relationship to explore. You can use data from the census (such as age and income), or data you collect yourself (such as number of ice cubes in a glass and melting time). For more sources and ideas, go to www.keymath.com/DG .

Collect pairs of data points. Use Fathom to graph your points and to find the line of best fit. Write a summary of your results.